Axial properties of N and N* resonances

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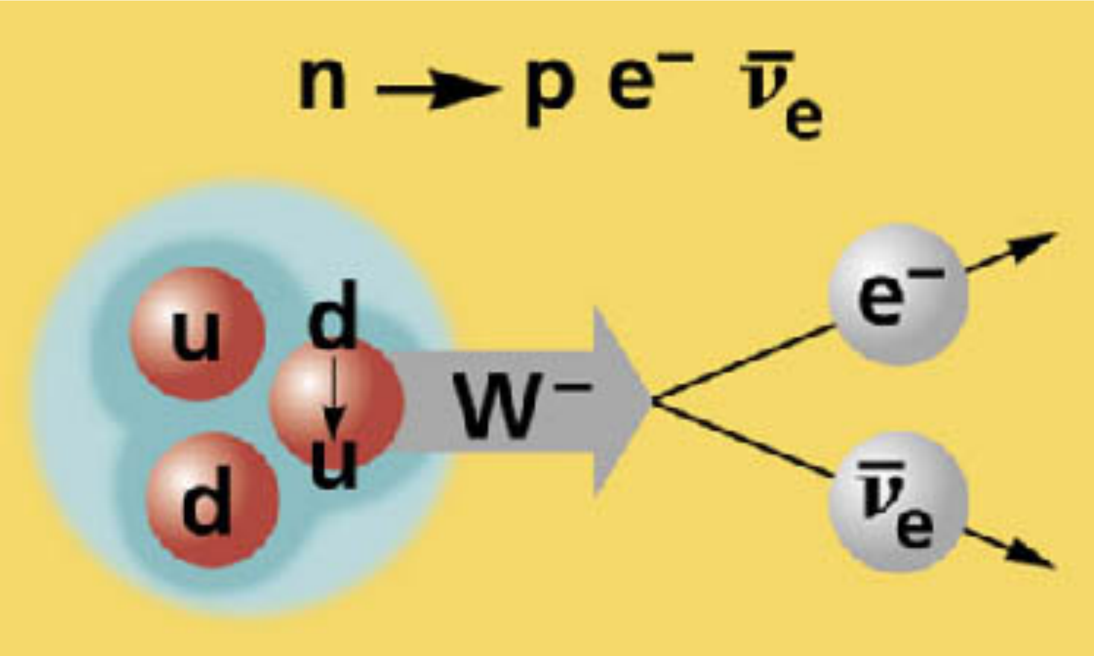
Weak Transitions



Decay parameter of n

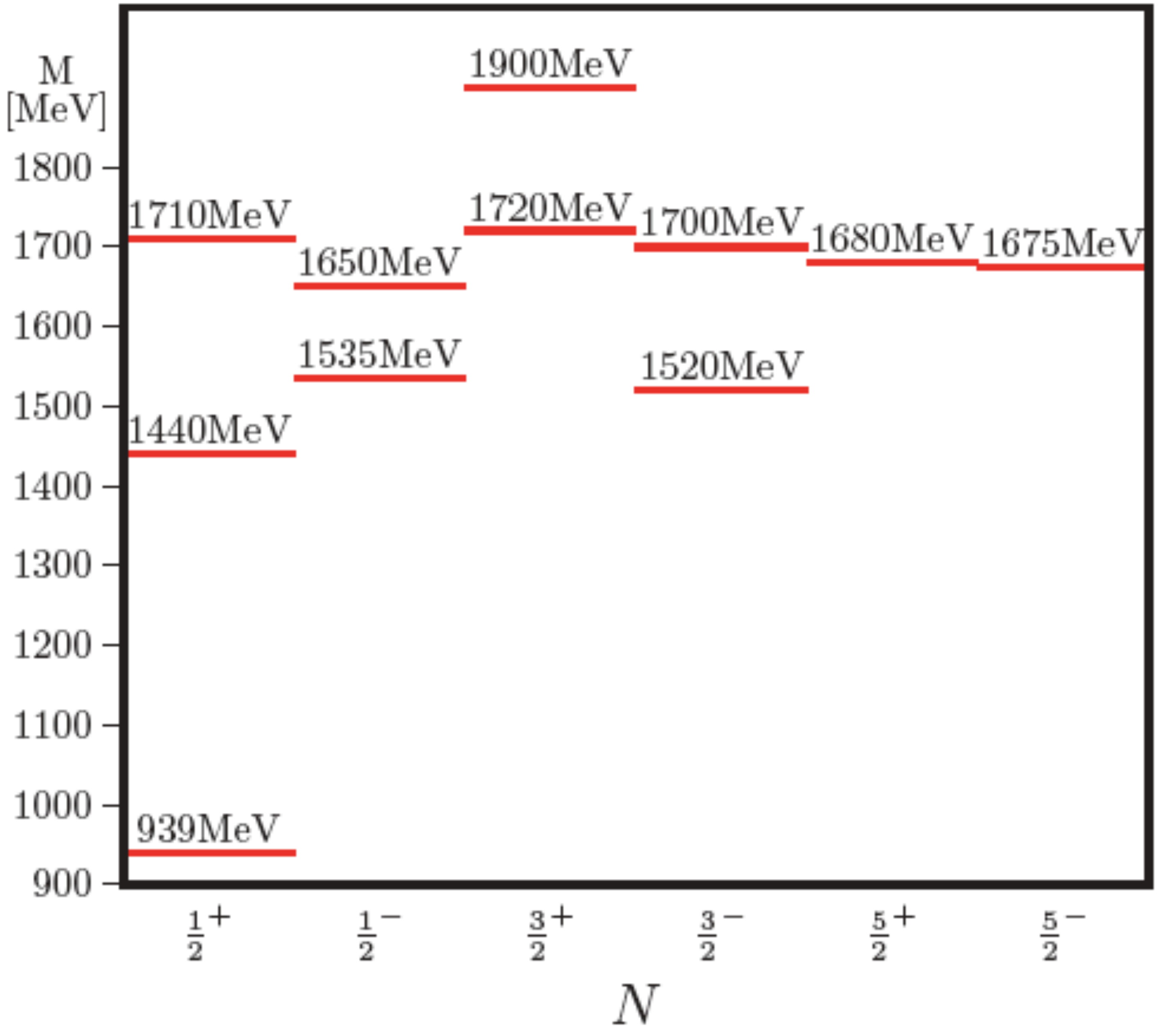
$$\lambda \equiv g_A/g_V = -1.2694 \pm 0.0028$$

Particle Data Group (PDG) 2010

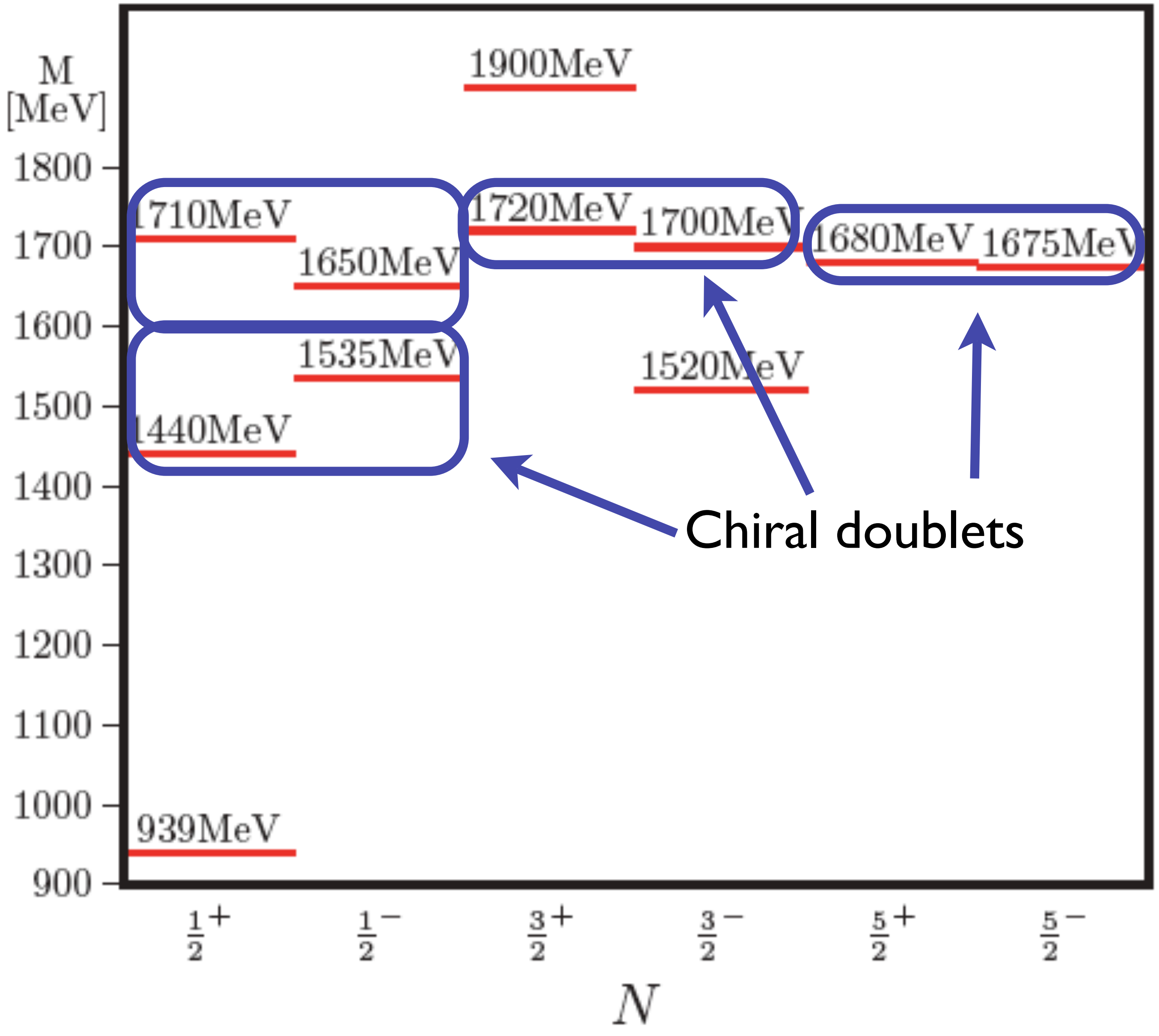
How about resonant states N*?

Recently, attracted much interest in studies of the strong interactions (QCD).

Chiral Restoration Scenario of QCD

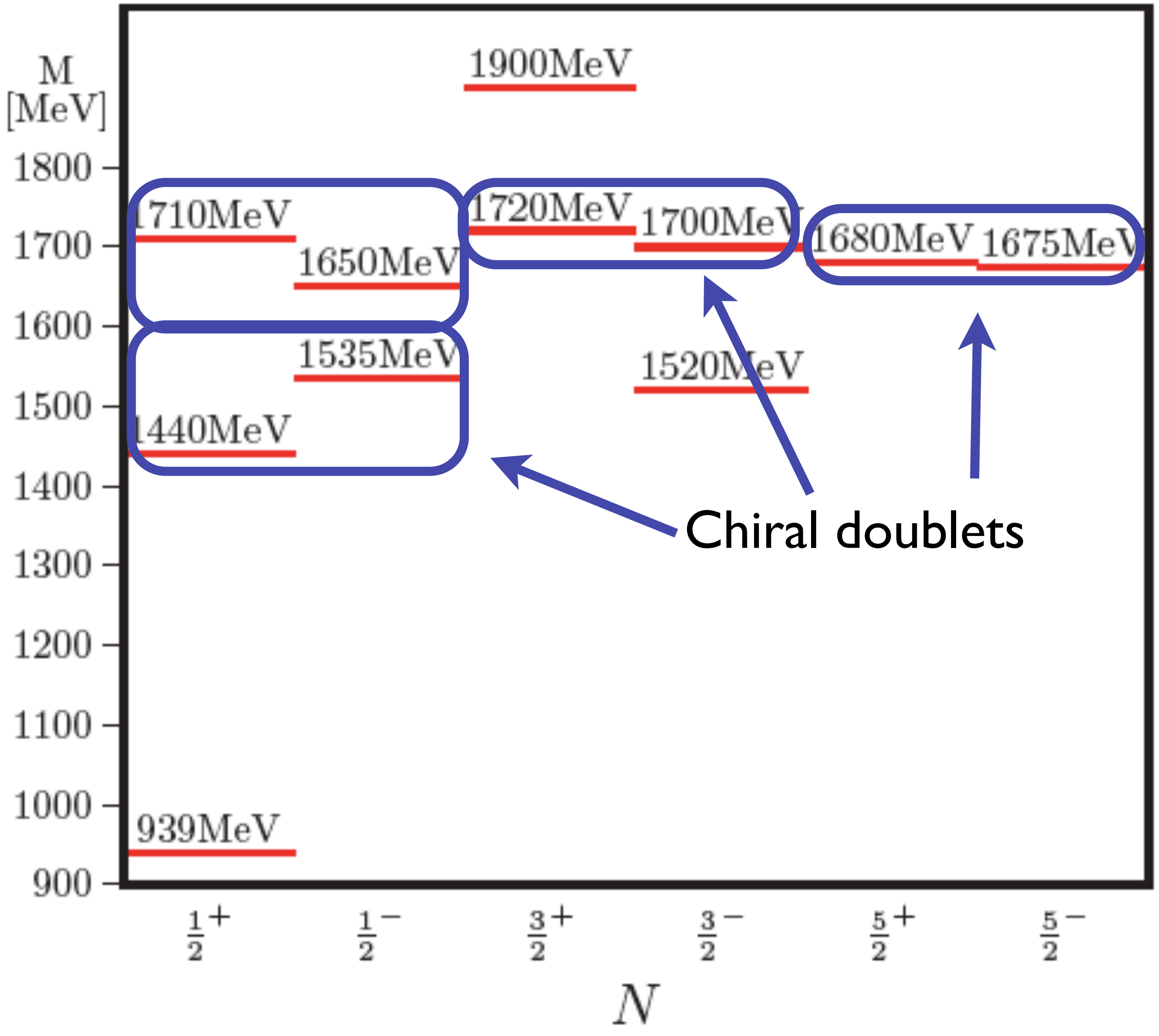


Chiral Restoration Scenario of QCD



Chiral doublets

Chiral Restoration Scenario of QCD



Chiral doublets

Goldberger-Treiman Relation

$$g_{\pi NN} = \frac{g_A M_N}{f_\pi}$$

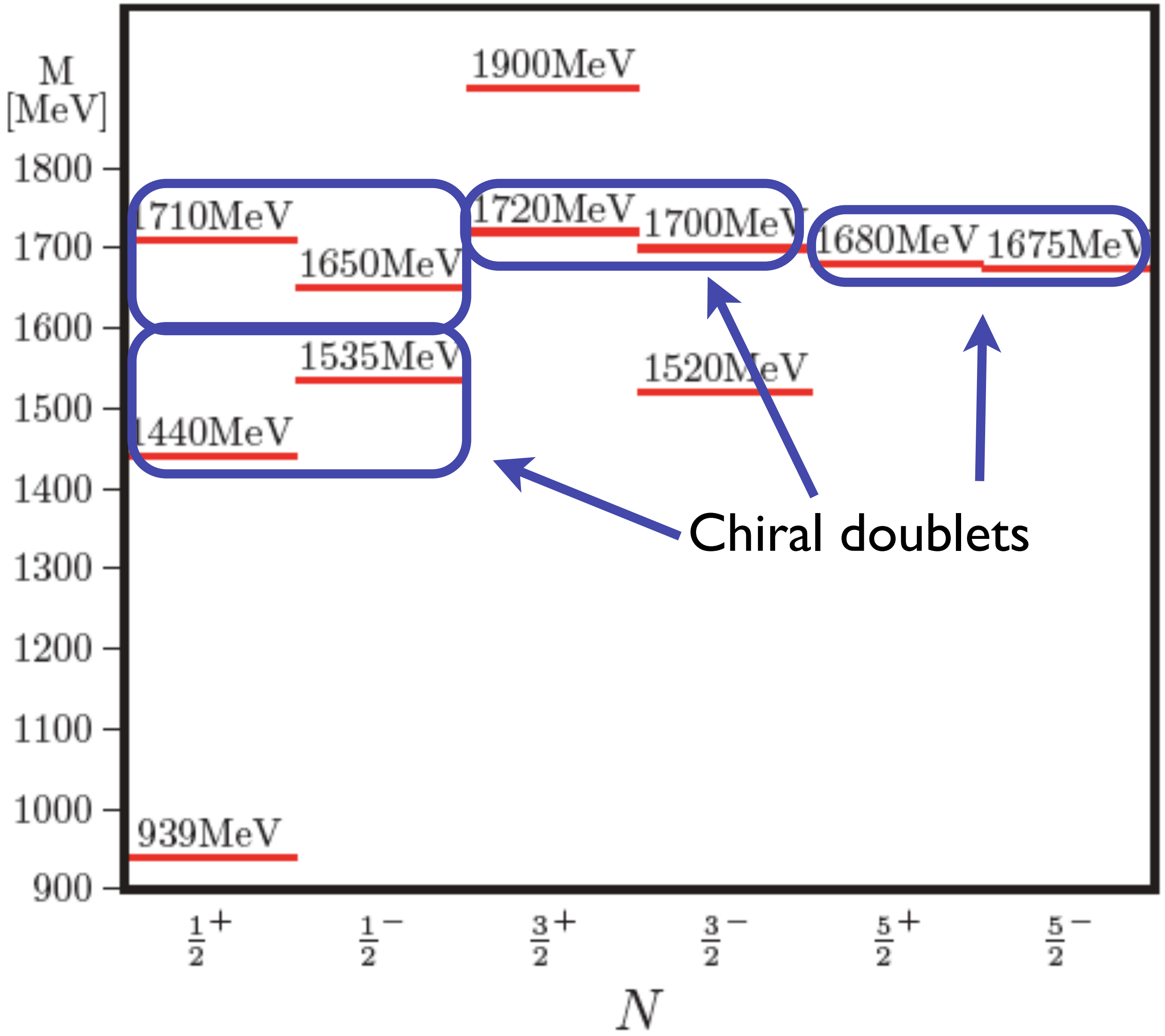
Axial charge

Nucleon mass

Pion Nucleon coupling

Pion decay constant

Chiral Restoration Scenario of QCD



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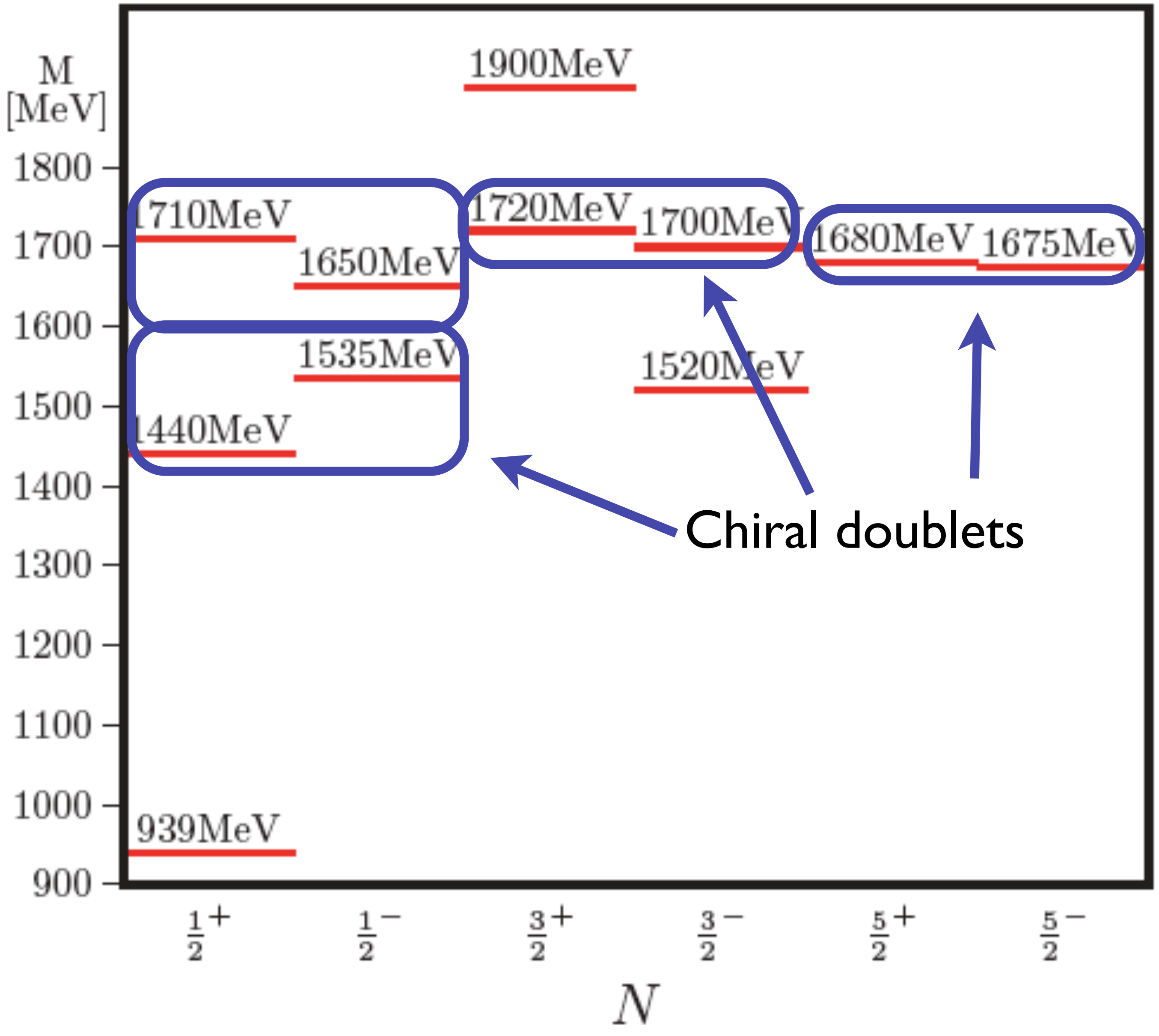
Restoration of the chiral symmetry :

$$g_{\pi NN} \simeq 0$$

L. Ya. Glozman: Phys. Rev. Lett. 99, 191602 (2007)

Chiral Restoration Scenario of QCD



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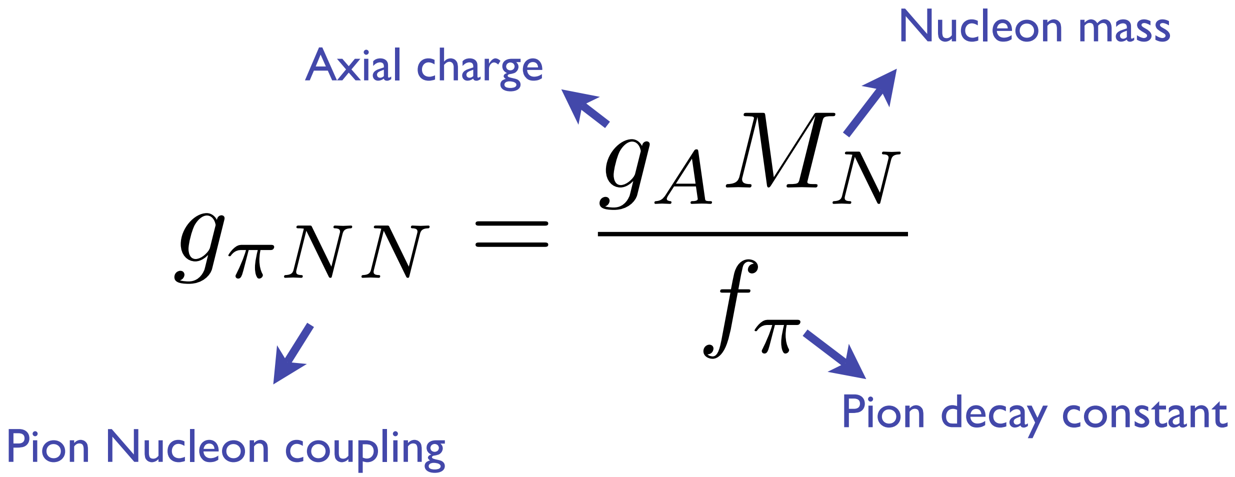

Axial charge

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$$g_{\pi NN} = \frac{g_A M_N}{f_\pi}$$

Pion Nucleon coupling

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Restoration of the chiral symmetry :

$$g_{\pi NN} \simeq 0$$

L. Ya. Glozman: Phys. Rev. Lett. 99, 191602 (2007)

→ Requirement for restoration : $g_A \simeq 0$

Analytic and Lattice Results

- Non-relativistic SU(6) wave function

$$\frac{1}{2}^{+}, N(939): \quad G^A = \frac{5}{3}$$ ~ 1.66

$$\frac{1}{2}^{+}, N(1440): \quad G^A = \frac{5}{3}$$ ~ 1.66

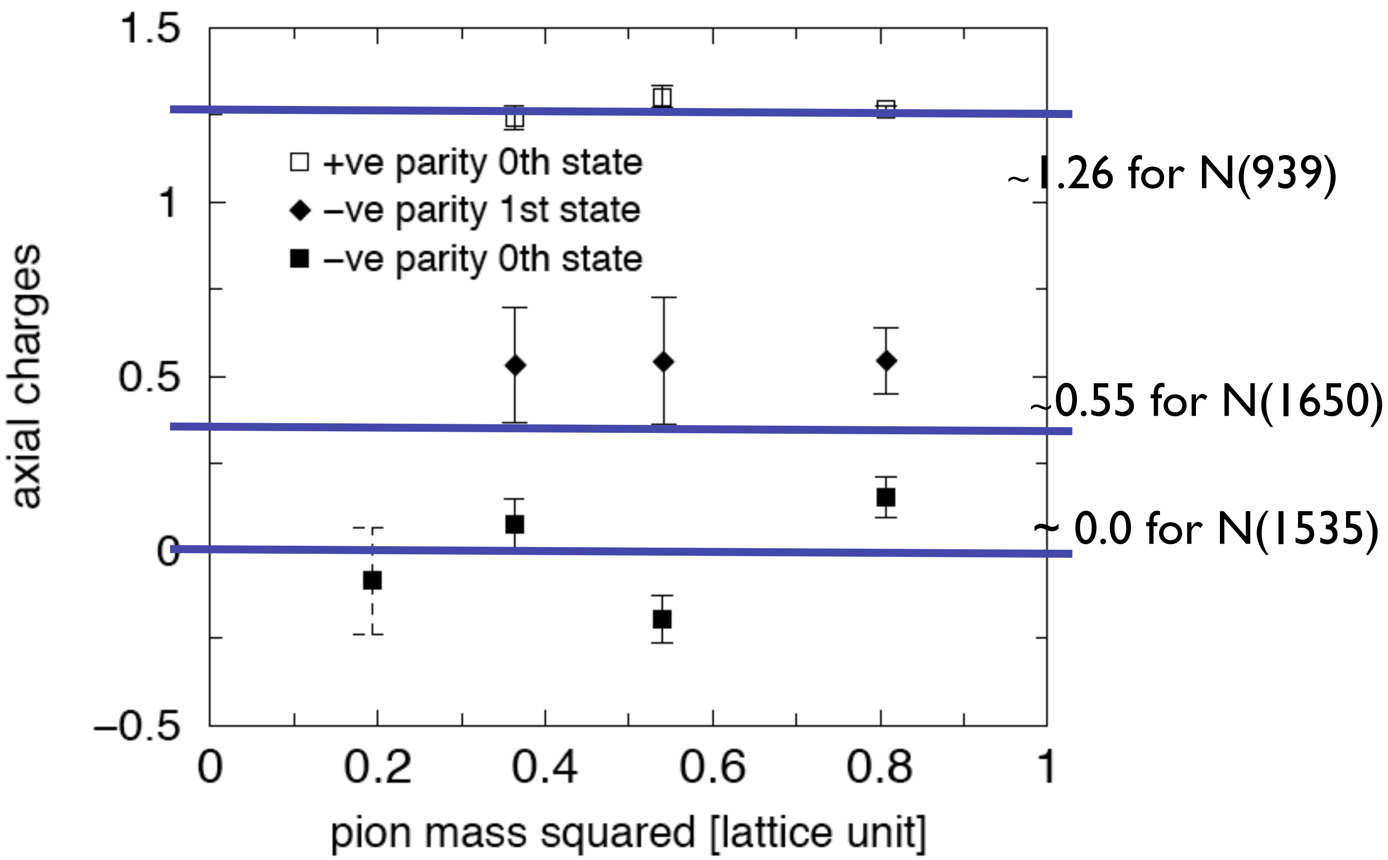
$$\frac{1}{2}^{+}, N(1710): \quad G^A = \frac{1}{3}$$ ~ 0.33

$$\frac{1}{2}^{-}, N(1535): \quad G^A = -\frac{1}{9}$$ ~ -0.11

$$\frac{1}{2}^{-}, N(1650): \quad G^A = \frac{5}{9}$$ ~ 0.55

Small

- Lattice results



L.Ya. Glozman, A.V. Nefediev: Nucl. Phys. A 807, 38 (2008)

Toru T. Takahashi and Teiji Kunihiro: Phys. Rev. D 78, 011503 (2008)

Relativistic Constituent Quark Model

- Framework: Relativistic Quantum Mechanics (RQM)

i.e. quantum theory respecting Poincaré invariance

(theory on $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

- Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int} = \sqrt{\hat{H}_0^2 - \hat{\vec{P}}^2_{free}} + \sum_{i<j} \left[\hat{V}_{ij}^{conf} + \hat{V}_{ij}^{hf} \right]$$

Eigenvalue equation : $\hat{M}|P, J, \Sigma\rangle = m|P, J, \Sigma\rangle$

Relativistic kinetic energy : $H_0 = \sum_{i=1}^{3} \sqrt{\vec{p}_i^2 + m_i^2}$

Confinement : $\hat{V}_{ij}^{conf} = V_0 + C r_{ij}$

Hyperfine interaction $\hat{V}_{ij}^{hf}$: OGE, psGBE, EGBE

Axial Form Factors

- Matrix elements of the axial current operator

$$\left\langle P,\frac{1}{2},\Sigma'\middle|\hat{A}^\mu_a\middle|P,\frac{1}{2},\Sigma\right\rangle = \bar{U}(P,\Sigma')g_A\gamma^\mu\gamma_5\frac{\tau_a}{2}U(P,\Sigma)$$

$$\left\langle P,\frac{3}{2},\Sigma'\middle|\hat{A}^\mu_a\middle|P,\frac{3}{2},\Sigma\right\rangle = \bar{U}^\nu(P,\Sigma')g_A\gamma^\mu\gamma_5\frac{\tau_a}{2}U_\nu(P,\Sigma)$$

$$\left\langle P,\frac{5}{2},\Sigma'\middle|\hat{A}^\mu_a\middle|P,\frac{5}{2},\Sigma\right\rangle = \bar{U}^{\nu\lambda}(P,\Sigma')g_A\gamma^\mu\gamma_5\frac{\tau_a}{2}U_{\nu\lambda}(P,\Sigma)$$

g_A : Axial charge of N and N*

Incoming / Outgoing baryon states : $|V,M,J,\Sigma\rangle = |P,J,\Sigma\rangle$

$$\left\langle V,M,J,\Sigma\middle|\hat{A}^\mu_a\middle|V,M,J,\Sigma\right\rangle = \frac{2}{MM'}\sum_{\sigma_i\sigma'_i}\sum_{\mu_i\mu'_i}\int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}'_2 d^3\vec{k}'_3$$

$$\times\sqrt{\frac{\left(\sum_i\omega'_i\right)^3}{\prod_i 2\omega'_i}}\prod_{\sigma'_i}D^{*\frac{1}{2}}_{\sigma'_i\mu'_i}\left\{R_W[k'_i;B(V')]\right\}\Psi^*_{M'J'\Sigma'}\left(\vec{k}'_1,\vec{k}'_2,\vec{k}'_3;\mu'_1,\mu'_2,\mu'_3\right)\boxed{\left\langle p'_1,p'_2,p'_3;\sigma'_1,\sigma'_2,\sigma'_3\middle|\hat{A}^\mu_{a,rd}\middle|p_1,p_2,p_3;\sigma_1,\sigma_2,\sigma_3\right\rangle}$$

$$\times\sqrt{\frac{\left(\sum_i\omega_i\right)^3}{\prod_i 2\omega_i}}\prod_{\sigma_i}D^{\frac{1}{2}}_{\sigma_i\mu_i}\left\{R_W[k_i;B(V)]\right\}\Psi_{MJ\Sigma}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3;\mu_1,\mu_2,\mu_3\right)\quad 2MV_0\delta^3\left(M\vec{V}-M'\vec{V}'-\vec{q}\right)$$

where $p_i = B_c(V)k_i,\ p'_i = B_c(V')k'_i$ and $w_i = \sqrt{\vec{k}_i^2+m_i^2}$

Point Form Spectator Model

- Point form spectator model

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \left| \hat{O}_{rd} \right| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$

$$= 3N \left\langle p_1', \sigma_1' \left| \hat{O}_{spec} \right| p_1, \sigma_1 \right\rangle 2p_{20} \delta\left(\vec{p}_2 - \vec{p}_2'\right) 2p_{30} \delta\left(\vec{p}_3 - \vec{p}_3'\right) \delta_{\sigma_2 \sigma_2'} \delta_{\sigma_3 \sigma_3'}$$

- Axial current

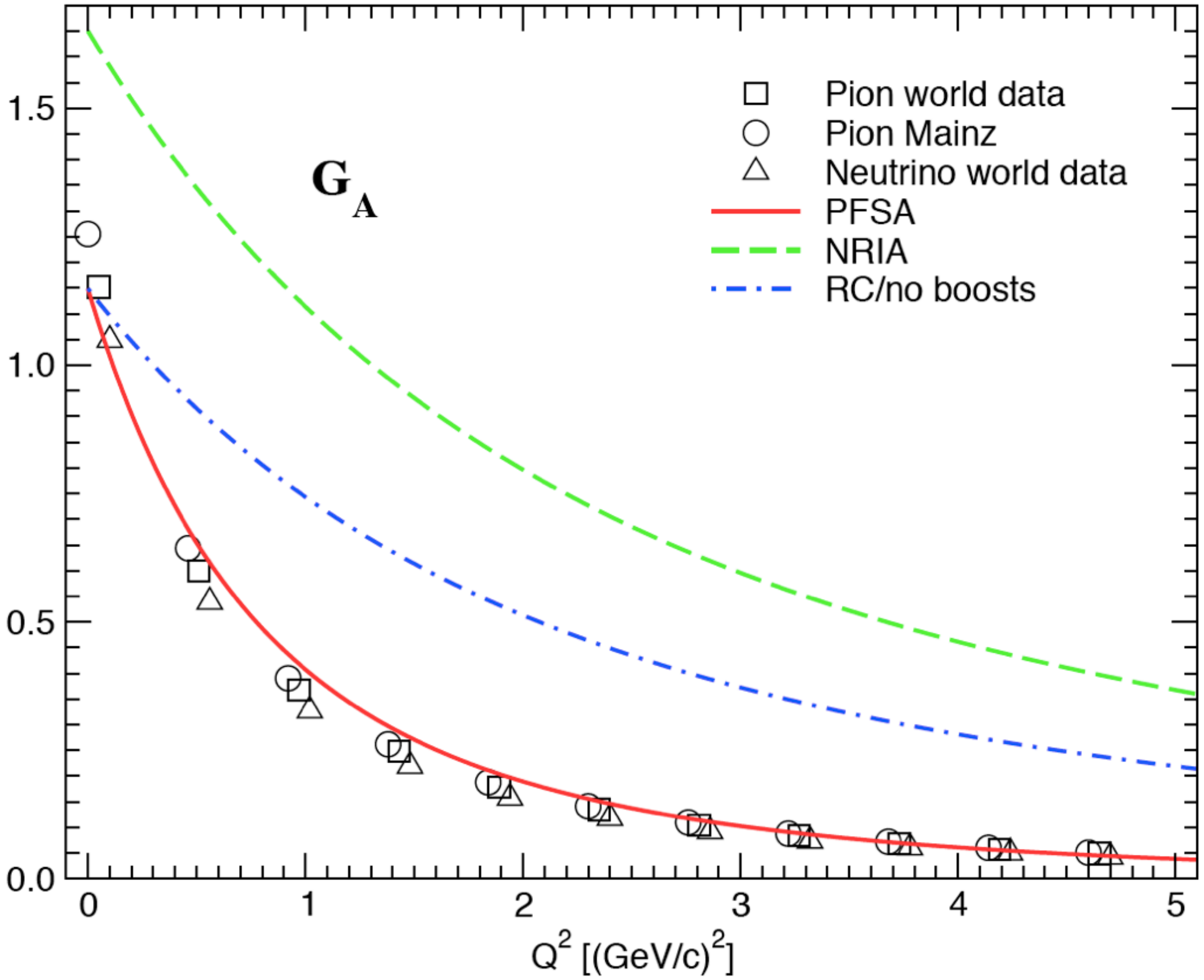
$$\langle p_1', \sigma_1' | \hat{A}^{\mu}_{a,spec} | p_1, \sigma_1 \rangle = \bar{u}(p_1', \sigma_1') \left[g_A^q \gamma^\mu + \frac{2 f_\pi}{\tilde{Q}^2 + m_\pi^2} g_{qq\pi} \tilde{q}^\mu \right] \gamma_5 \frac{\tau_a}{2} u(p_1, \sigma_1)$$

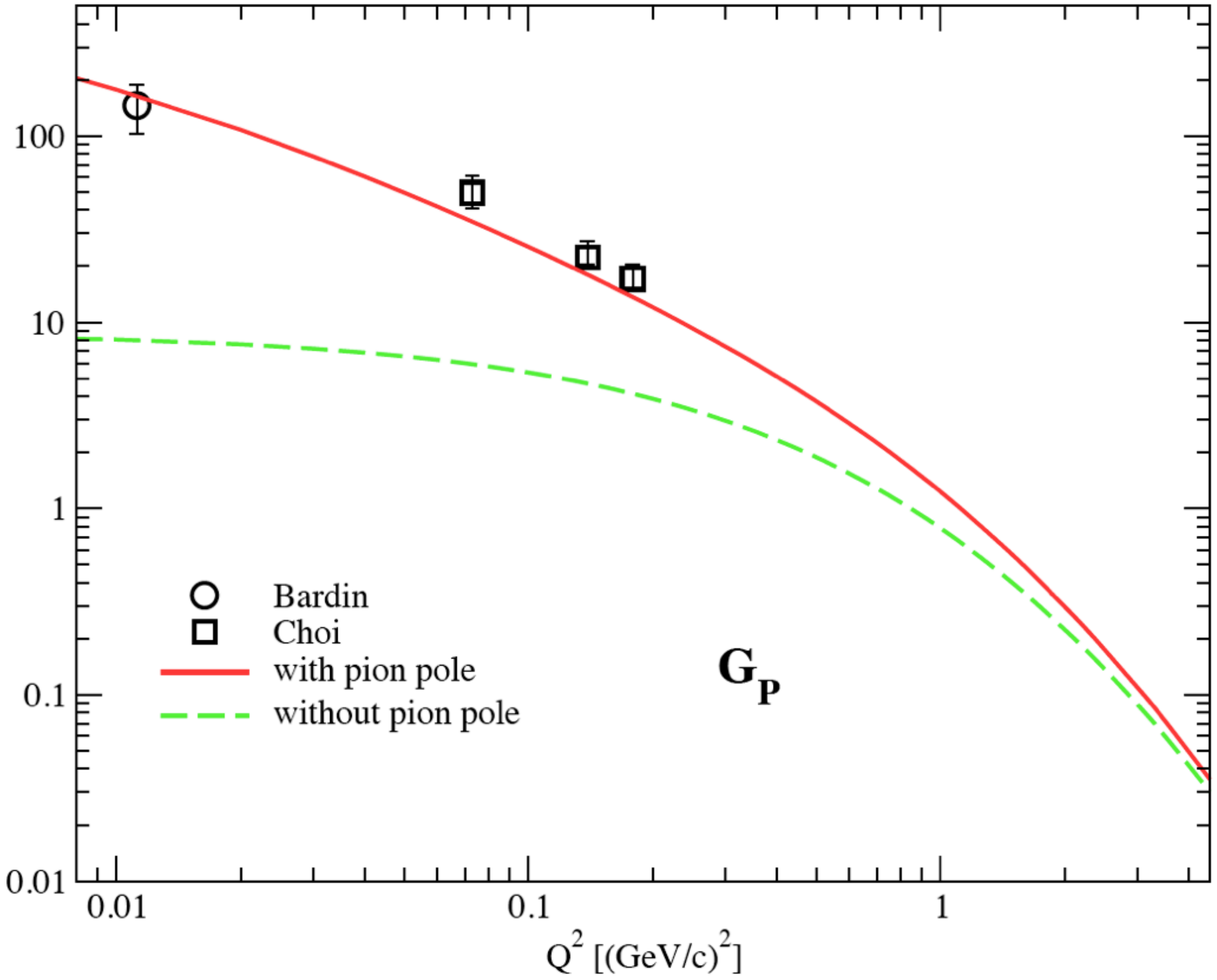
$g_A^q = 1$ (pointlike constituent quark)

Results

- Axial form factors of N
- Axial charges of N and some N* relative to other results
- Axial charges of all N* resonances below 2 GeV
- Axial form factors of N*

Axial and Pseudo-scalar Form Factors of nucleon





$$g_A^{EXP} = 1.2695 \pm 0.0029$$

$$g_A^{psGBE} = 1.15$$

L.Ya. Glozman, M.Radici, R.F.Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B 516, 183 (2001)

Comparison

State	J^P	EGBE[1]	Lattice QCD[3]	GN[2]	NR
$N(939)$	$\frac{1}{2}^+$	1.15	1.10∼1.40	1.66	1.65
$N^*(1440)$	$\frac{1}{2}^+$	1.16	–	1.66	1.61
$N^*(1535)$	$\frac{1}{2}^-$	0.02	∼0.00	-0.11	-0.20
$N^*(1710)$	$\frac{1}{2}^+$	0.35	–	0.33	0.42
$N^*(1650)$	$\frac{1}{2}^-$	0.51	∼0.55	0.55	0.64

1. K. S. Choi, W. Plessas, R. F. Wagenbrunn : Phys. Rev. C 81, 028201 (2010)
2. L. Ya. Glozman, A. V. Nefediev: Eur. Phys. J. A 12, 91 (2001)
3. Toru T. Takahashi and Teiji Kunihiro : Phys. Rev. D 78, 011503 (2008) and further lattice QCD calculation

Axial Charges of Chiral Partners

		EGBE		psGBE		OGE	
State	J^P	Mass	g_A	Mass	g_A	Mass	g_A
$N(939)$	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
$N^*(1520)$	$\frac{3}{2}^-$	1524	-0.64	1519	-0.21	1520	-0.15
$N^*(1440)$	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
$N^*(1535)$	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
$N^*(1680)$	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70
$N^*(1675)$	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80
$N^*(1710)$	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
$N^*(1650)$	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
$N^*(1720)$	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
$N^*(1700)$	$\frac{3}{2}^-$	1608	-0.10	1647	-0.50	1690	-0.47

Classification

$(LS)J^P$				
$(0\,\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
$(0\,\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\mathbf{\Xi(1690)}^{100}$
$(0\,\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
$(1\,\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\mathbf{\Sigma(1560)}^{94}$	
$(1\,\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\mathbf{\Sigma(1620)}^{100}$	
$(1\,\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
$(1\,\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\mathbf{\Sigma(1940)}^{100}$	
$(1\,\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\mathbf{\Sigma(1950)}^{100}$

$(LS)J^P$				
$(0\,\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
$(0\,\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\mathbf{\Sigma(1690)}^{99}$		
$(1\,\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\mathbf{\Sigma(1750)}^{94}$		
$(1\,\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			

K. S. Choi, W. Plessas, R. F. Wagenbrunn : Phys. Rev. D 82, 014007 (2010)

T. Melde, W. Plessas and B. Sengl: Phys. Rev. D 77, 114002 (2008) and Particle Data Group (PDG) 2010

Classification

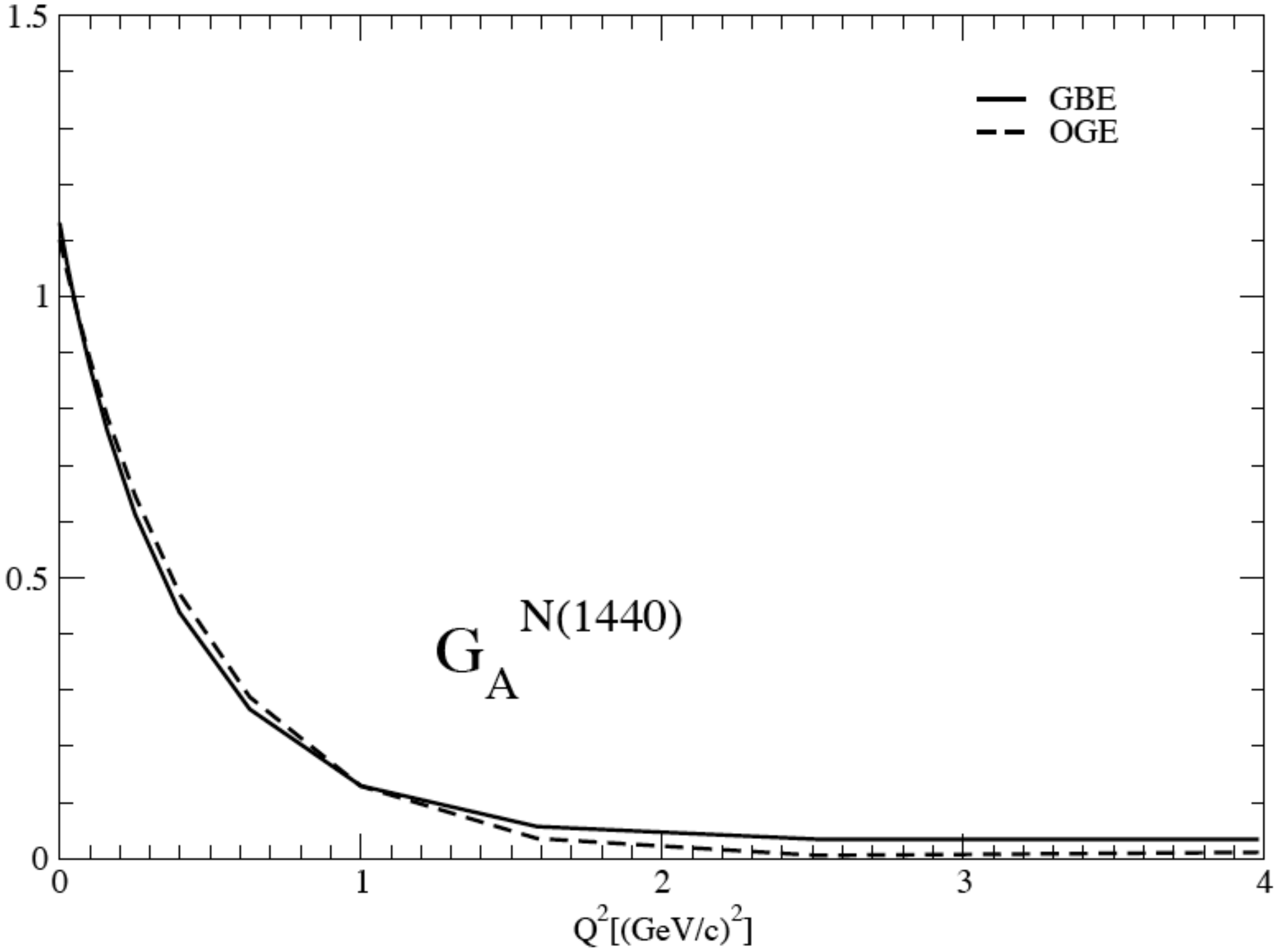
$(LS)J^P$					
$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$	
$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\mathbf{\Xi(1690)}^{100}$	
$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$	0.02	$\Sigma(1880)^{99}$		
$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\mathbf{\Sigma(1560)}^{94}$		-0.15
$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\mathbf{\Sigma(1620)}^{100}$		
$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$	
$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\mathbf{\Sigma(1940)}^{100}$		
$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\mathbf{\Sigma(1950)}^{100}$	

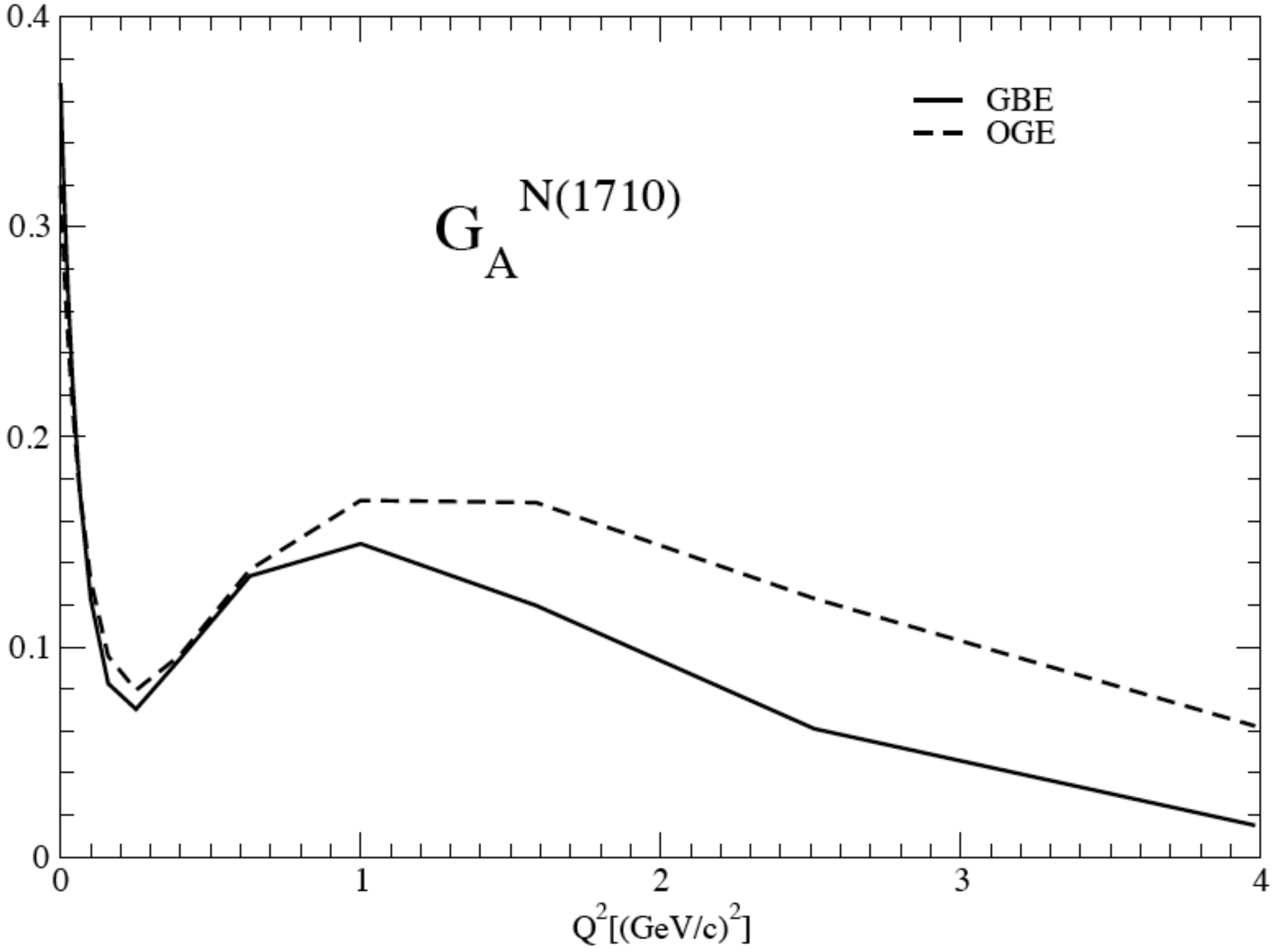
$(LS)J^P$				
$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\mathbf{\Sigma(1690)}^{99}$		
$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\mathbf{\Sigma(1750)}^{94}$	-0.08	
$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$	-0.76		

K. S. Choi, W. Plessas, R. F. Wagenbrunn : Phys. Rev. D 82, 014007 (2010)

T. Melde, W. Plessas and B. Sengl: Phys. Rev. D 77, 114002 (2008) and Particle Data Group (PDG) 2010

Axial Form Factors of N(1440) and N(1710)

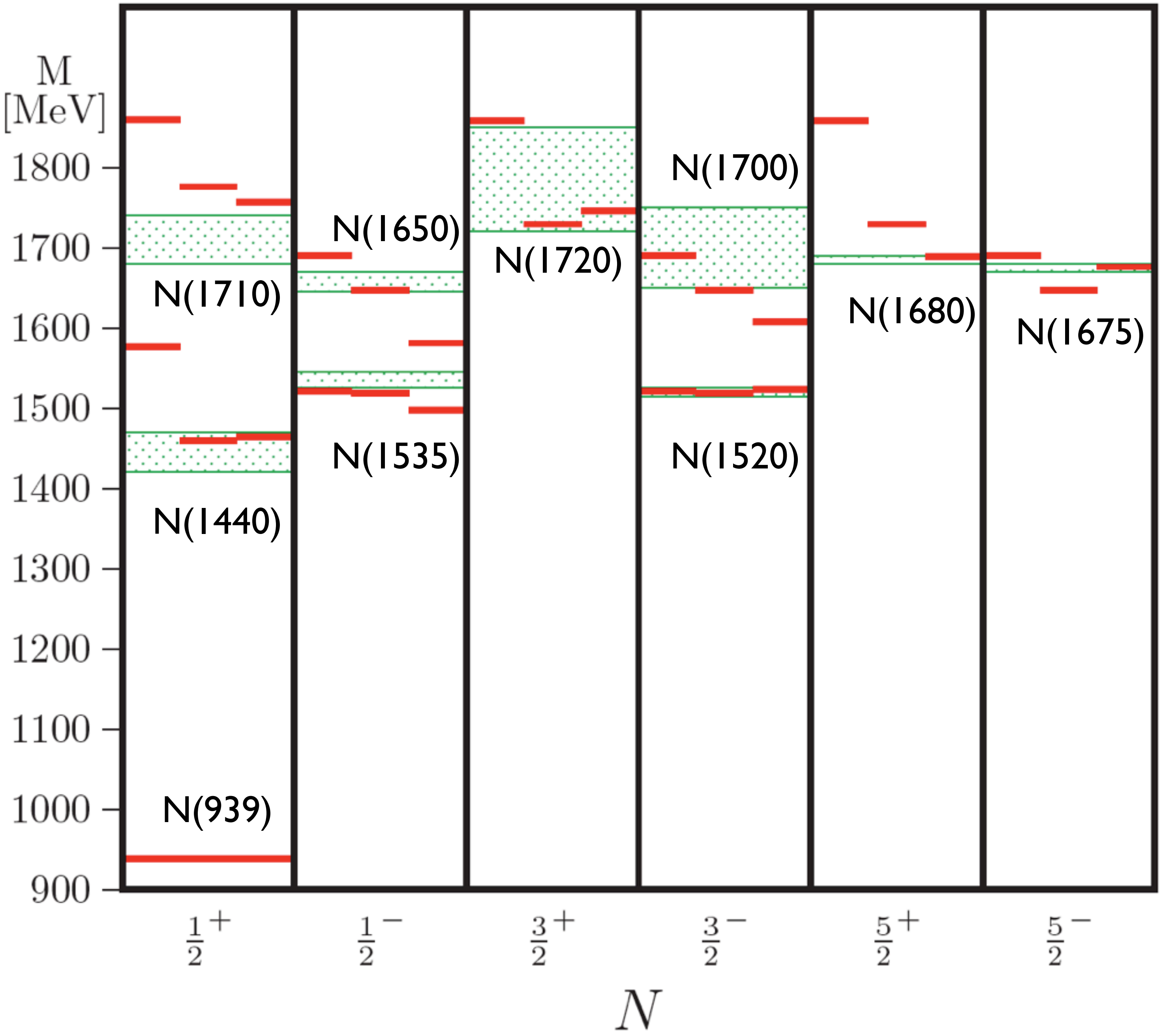




Summary

- Performed a consistent study of g_A for N and N* within the RCQMs.
- Covariant predictions of RCQMs show generally considerable relativistic effects and describe experimental data well.
- In cases, where no experiments exist, there is reasonable agreement with lattice QCD results.
- The issue regarding chiral symmetry restoration remains open; the sizes of the g_A for baryon resonances are consistent with a recent classification into flavor multiplets.
- It will be interesting to extend these relativistic studies to electromagnetic and weak transition form factors.

Nucleon Spectra



Left levels - OGE : L.Theussl, R.F.Wagenbrunn, B. Desplanques, and W. Plessas: Eur. Phys. J. A 12, 91 (2001)
Middle levels - psGBE : L.Ya. Glozman, W. Plessas, K.Varga, and R.F.Wagenbrunn: Phys. Rev. D 58, 094030 (1998)
Right levels - EGBE : K.Gantschnig, R.Kainhofer, W. Plessas, B. Sengl, and R.F.Wagenbrunn: Eur. Phys. J. A 23, 507 (2005)
Green zones - Experimental data in PDG (2010)